Lower Sixth AL Maths Homework – Week 7/8 **Name:** Answers

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

If $(x+2)$ is a factor then $f(-2) = 0$

$f(-2) = 2(-2)^3 - 5(-2)^2 + a(-2) + a = 0$

$-16 - 20 - 2a + a = 0$

$\therefore a = -36$

4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

If $(x-3)$ is a factor then $f(3) = 0$

$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$

$\therefore (x-3)$ is a factor of $f(x)$.

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

Comparing coefficients:

(4)

$f(x) = (x-3)(4x^2 + bx + 2) = 4x^3 - 12x^2 + 2x - 6$

$bx^2 - 12x^2 = -12x^2$

$\therefore b - 12 = -12$

$b = 0$

$\therefore f(x) = (x-3)(4x^2 + 2)$

$a = 4$
$b = 0$
$c = 2$

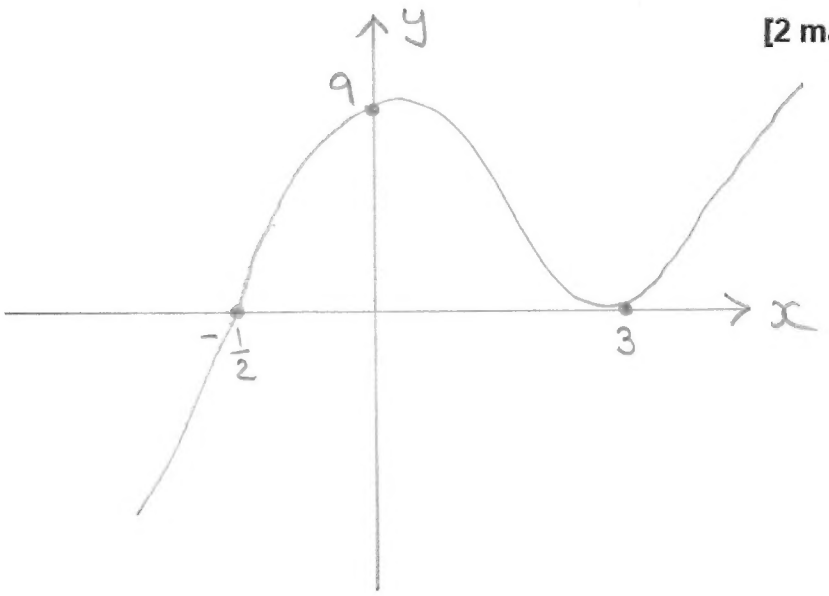
$b^2 - 4ac = 0^2 - 4(4)(2) = -32 < 0$

$\therefore 4x^2 + 2$ has no real roots

hence $x = 3$ is the only real root for $f(x)$.

2 Sketch the graph of the curve with equation $y = (2x+1)(x-3)^2$

root at $x = -\frac{1}{2}$
repeated root at $x = 3$
shape ∩
y-intercept =
$1 \times -3 \times -3 = 9$



[2 marks]

8 (a) Find the first **three** terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$

[3 marks]

$${}^{10}C_0 (-2x)^0 (1)^{10} + {}^{10}C_1 (-2x)^1 (1)^9 + {}^{10}C_2 (-2x)^2 (1)^8 + \ldots$$

$$= 1 - 20x + 180x^2 + \ldots$$

8 (b) Carly has lost her calculator. She uses the first three terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$ to evaluate 0.998^{10}

Find Carly's value for 0.998^{10} and show that it is correct to **five** decimal places.

[3 marks]

$(1-2x)^{10} = 0.998^{10}$

$1 - 2x = 0.998$

$0.002 = 2x$

$\therefore x = 0.001$

$1 - 20(0.001) + 180(0.001)^2 = 0.98018$

$0.998^{10} = 0.9801790434$

$= 0.98018$ (5dp).

This matches Carly's value.

__ / 17